

## Any Typewriter can Type Scientific Symbols!

### TECHNICAL, MATHEMATICAL, GREEK LETTER OR CUSTOM SYMBOLS

A simple adapter and Typit symbol elements are all you need to professionally type symbols along with your regular typing. No matter what make of typewriter you have, you can transform it into a limitless typing machine!

No more leaving space for symbols . . . no more inking in symbols . . . no more using improper typewriter letters in place of the proper symbols. Send today for FREE catalogs on Typit stock symbols and custom symbol designing. Specify make and type of typewriter. Write to:

**MECHANICAL ENTERPRISES, INC.**

3127 Colvin Street, Alexandria, Va. 22314

## NUMBER THEORY

Edited by **W. J. LeVeque & E. G. Straus**

Proceedings of Symposia in Pure Mathematics,  
Volume 12

112 Pages; List Price \$5.60; Member Price \$4.20

At the seventy-third annual meeting of the American Mathematical Society, Houston, Texas, January 24-28, 1967, a Special Session on recent advances in the theory of numbers was held. The Program Committee for the Special Session consisted of E. G. Straus and W. J. LeVeque. The articles in this volume comprise all but one of the papers presented. All of the talks were of 20 minutes duration except that of H. M. Stark, which was an hour address. In several cases, what was only sketched in the talk, because of lack of time, is presented in this volume in detail.

For additional information or to place an order  
please write:

**AMERICAN MATHEMATICAL SOCIETY**

P. O. Box 6248, Providence, Rhode Island 02904

Announcing a new mathematical book series . . .

## SIAM-AMS Proceedings

The PROCEEDINGS OF SYMPOSIA IN APPLIED MATHEMATICS has been published by the Society since 1949, the topics of the symposia being chosen by the AMS Committee on Applied Mathematics. In 1966, this committee became a joint committee with the Society for Industrial and Applied Mathematics. It, therefore, seemed appropriate to discontinue the PSAM series with Volume 19 and replace it with a new series to be known as the SIAM-AMS PROCEEDINGS. The new series will be hard-bound and, except for a newly designed cover, will be similar to the old series.

Volume 1 of the SIAM-AMS PROCEEDINGS is entitled Transport Theory. It consists of 336 pages and will sell for the list price of \$11.00 and a member price of \$8.25.

This first volume of the new series will be a compilation of papers presented at a symposium in applied mathematics that was held in New York City on April 5-6, 1967; the editors are Richard Bellman, Garrett Birkhoff, and Ibrahim Abu-Shumays.

The book's four parts provide an excellent and comprehensive survey of the current state of transport theory and should serve as a stimulus for research in this field.

**AMERICAN MATHEMATICAL SOCIETY**

P. O. Box 6248, Providence, Rhode Island

# PRENTICE-HALL BOOKS

## **GREENSPAN**

- **Lectures on the Numerical Solution of Linear, Singular, and Non-singular Differential Equations**—by Donald Greenspan, University of Wisconsin.

Applies the capabilities of digital computers to scientific and technological problems. Develops boundary value techniques utilizing finite differences that can be programmed for any digital computer. Scientists and technologists are provided with a complete survey of the latest methods in the field. Bibliographical references inserted throughout the text allow the teacher to adapt the material for classroom presentation. 1968, 185 pp., \$6.95 (52749-9)

## **FIKE**

- **Computer Evaluation of Mathematical Functions**—by C. T. Fike, IBM Systems Research Institute.

This text describes mathematical methods used to code computer programs for evaluating mathematical functions. Material previously available only in journal articles are included in this book. Suitable for college juniors and seniors in mathematics and engineering, and provides useful techniques for the professional. Contents include: Error in Function Evaluation Computations. Square-Root and Cube-Root Evaluation. Reducing the Argument Range. Polynomial Evaluation Methods. and more. 1968, 240 pp., \$10.50 (16572-0)

## **DESMONDE**

- **A Conversational Graphic Data Processing System: The IBM 1130/2250**—by William H. Desmond, Nassau Community College.

An introduction to the programming of a conversational graphic data processing system, this book presents the elementary coding for the IBM 1130/2250 system, along with the basics of Light Pen usage. Essentially, this book is a programming manual for the 2250 display scope, attached to an IBM 1130 computer. Contains photographs of displays possible with the equipment, charts, review questions for each chapter, and suggestions for further reading. 1968, 186 pp., \$10.95 (17194-2)

## **TOMPKINS-WILSON**

- **Elementary Numerical Analysis**—by Charles B. Tompkins, University of California, Los Angeles, and Walter L. Wilson, Jr., University of Alabama.

The authors use a system of computing numbers to demonstrate that one can only expect to compute estimates to zeros of linear and quadratic polynomials. The text is written in a one-line format, much like FORTRAN. This one-line format, containing the symbols, capitals and roman numbers of the computer output, makes its descriptive material and equations readily understandable in both the computer laboratory and the classroom. Partial contents: Taylor's Formula. Truncation Error. Iteration Process. Newton's Method. System of Linear Equations. Eigenvalues and Eigenvectors. July 1969, approx. 400 pp., \$10.50 (25942-4)

## **ARBIB**

- **Theories of Abstract Automata**—by Michael A. Arbib, Stanford University.

The first advanced text to treat in depth the mathematical investigation of the general questions raised by the study of information processing systems, be they men or machines. Particularly focusing on algebraic and combinatorial problems so raised. Part I—gives perspective, with an overview of automata theory, and outlines basic tools used in automata theory today. Part II—Serves as a complete text of basic automata theory. Part III—Introduces reader to partial recursive functions, and gives an account of topics now at the forefront of research. August 1969, approx. 416 pp., \$14.95 (91336-8)

## **GORDON**

- **Systematic Simulation**—by Geoffrey Gordon, IBM New York Scientific Center.

An introduction to the principles of simulation and the application of several simulation languages to system studies. No specialized knowledge in any particular discipline is assumed, although familiarity with some computer programming and Fortran is necessary. Contents: System Models, System Simulation. Continuous System Simulation. Industrial Dynamics. The Dynamo-Programming Language. Probability Concepts in Simulation. Arrival Patterns and Service Times. Discrete System Simulation. Simulation with Fortran. Simulation Programming Techniques. Introduction to GPSS. GPSS Examples. Introduction to SIMSCRIPT. Management of Sets in SIMSCRIPT. The System Evaluation and Analysis Language (SEAL). Verification of Simulation Results: August 1969, approx. 320 pp., \$11.50 (88180-5)

Orders will be processed faster if the title code and title appear on the order form.  
For approval copies, write: Box 903—

**PRENTICE-HALL, ENGLEWOOD CLIFFS, NEW JERSEY 07632**

# Mathematical Offprint Service

Perhaps you would be interested in subscribing to the AMS MATHEMATICAL OFFPRINT SERVICE (MOS). MOS has been in operation since July 1968 and is currently serving mathematicians throughout the world. It provides them with offprints and title listings of articles in their fields of interest at a relatively low cost. Subscribers are asked to submit an initial payment of not less than \$30 to begin their subscriptions, the price of an offprint is variable, ranging from \$0.25 to \$0.50. The exact price of each offprint depends on the size of the offprint and the price that MOS must pay to obtain it.

To obtain a pamphlet describing this service in detail, please complete the order form below. We will also send you a MOS profile form, list of participating journals and subject classification scheme.

*Please mail this form to:*

## **MATHEMATICAL OFFPRINT SERVICE**

American Mathematical Society  
P. O. Box 6248  
Providence, Rhode Island 02904

I wish to subscribe to the American Mathematical Society's  
**MATHEMATICAL OFFPRINT SERVICE.**

Name \_\_\_\_\_

Address \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

I would like to receive the MOS subject classification scheme in the following language (check one):

English  French  German  Italian  Russian

The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.

# Mathematics of Computation

## TABLE OF CONTENTS

JULY 1969

Some Computer-Assisted Topological Models of Hilbert Fundamental Domains . . . . .	HARVEY COHN	475
Calculation of Dirichlet $L$ -Functions . . . . .	ROBERT SPIRA	489
An Algorithm for the Determination of Space Groups . . . . .	HAROLD BROWN	499
On a Problem of Hasse. . . . .	H. ZASSENHAUS & J. LIANG	515
Computation of Minimal Length Full Steiner Trees on the Vertices of a Convex Polygon. . . . .	E. J. COCKAYNE	521
Calculation of the First Factor of the Class Number of the Cyclotomic Field . . . . .	TAUNO METSÄNKYLÄ	533
On Relatively Prime Odd Amicable Numbers . . . . .	PETER HAGIS, JR.	539
On Divisibility by Nine of the Sums of Even Amicable Pairs . . . . .	ELVIN LEE	545
An Iterative Finite-Difference Method for Hyperbolic Systems . . . . .	S. ABARBANEL & G. ZWAS	549
On Systems of Difference Equations with Wrong Boundary Conditions . . . . .	STANLEY OSHER	567
On the Approximate Minimization of Functionals . . . . .	JAMES W. DANIEL	573
Simultaneous Approximation of a Set of Bounded Real Functions . . . . .	J. B. DIAZ & H. W. McLAUGHLIN	583
A Method for Solving Nonlinear Volterra Integral Equations of the Second Kind . . . . .	PETER LINZ	595
Quadrature Methods Based on Complex Function Values. . . . .	J. N. LYNES	601
Osculatory Interpolation . . . . .	S. W. KAHNG	621
Rational Chebyshev Approximations for the Error Function . . . . .	W. J. CODY	631
Chebyshev Polynomial Expansion of Bose-Einstein Functions of Orders 1 to 10. . . . .	EDWARD W. NG, C. J. DEVINE & R. F. TOOPER	639
The Integral of the $N$ th Power of the Voigt Function . . . . .	ALEX REICHEL	645
Summation of a Slowly Convergent Series Arising in Antenna Study . . . . .	CHI FU DEN	651
On Computation of the Bivariate Normal Distribution . . . . .	D. E. AMOS	655
Gaussian Quadrature for the Integrals $\int_0^\infty \exp(-x^2)f(x)dx$ and $\int_0^b \exp(-x^2)f(x)dx$ . . . . .	N. M. STEEN, G. D. BYRNE & E. M. GELBARD	661
Notes on the Tables for Gaussian Quadrature of N. M. Steen, G. D. Byrne and E. M. Gelbard. . . . .	MILTON R. PINE	673
Gauss Quadrature Rules for the Evaluation of $2\pi^{-1/2} \int_0^\infty \exp(-x^2)f(x)dx$ . . . . .	DAVID GALANT	674
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS . . . . .		675
ARBIB 53, BARRON 54, BEYER, METROPOLIS & NEERGAARD 45, CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE, PARIS 41, COLE 56, FULLER 47, GALANT 42, HILDEBRAND 44, KARST 52, LAL & GILLARD 49, NEWMAN 50, SCHWARTZ 43, SHANKS & WRENCH 46, WEGNER 55, WESTERN & MILLER 51, WOLBERG 48		
TABLE ERRATA . . . . .		691
ABRAMOWITZ & STEGUN 442, HALL & SENIOR 443		
CORRIGENDUM . . . . .		693
RALSTON		

# THE INTEGRAL OF THE N<sup>th</sup> POWER OF THE VOIGT FUNCTION

Alex Reichel

## TABLE OF THE FUNCTION

$$\chi_n(t) = \int_{-\infty}^{+\infty} \{U_0(x,t)\}^n dx$$

where  $U_0(x,t)$  is the Voigt function

$$U_0(x,t) = \frac{1}{(4\pi t)^{1/2}} \int_{-\infty}^{+\infty} \frac{e^{-(x-y)^2/4t} dy}{1+y^2}$$

n	$x_n(t) \ t=1$	$x_n(t) \ t=2$	$x_n(t) \ t=3$	$x_n(t) \ t=4$
2	1.02993902E+00	8.57091445E-01	7.56923243E-01	6.88295035E-01
3	4.48167137E-01	3.01612821E-01	2.31663272E-01	1.89649611E-01
4	2.09899337E-01	1.13743427E-01	7.58311639E-02	5.58205655E-02
5	1.01956296E-01	4.44306276E-02	2.56964389E-02	1.70030375E-02
6	5.06354818E-02	1.77355462E-02	8.89613754E-03	5.29055596E-03
7	2.55279736E-02	7.18468374E-03	3.12520070E-03	1.67029214E-03
8	1.30104621E-02	2.94181830E-03	1.10960475E-03	5.32940818E-04
9	6.68560550E-03	1.21437053E-03	3.97160481E-04	1.71419813E-04
10	3.45771676E-03	5.04495349E-04	1.43060874E-04	5.54871142E-05
11	1.79760275E-03	2.10667460E-04	5.17965132E-05	1.80527083E-05
12	9.38542598E-04	8.83442050E-05	1.88327387E-05	5.89823003E-06
13	4.91777098E-04	3.71792733E-05	6.87168676E-06	1.93390345E-06
14	2.58465739E-04	1.56940467E-05	2.51489772E-06	6.35993252E-07
15	1.36198853E-04	6.64197943E-06	9.22789389E-07	2.09697654E-07
16	7.19335234E-05	2.81735587E-06	3.39362487E-07	6.92966193E-08
17	3.80676074E-05	1.19742274E-06	1.25050247E-07	2.29450348E-08
18	2.01811607E-05	5.09818601E-07	4.61600091E-08	7.61070600E-09
19	1.07156704E-05	2.17401840E-07	1.70657392E-08	2.52835570E-09
20	5.69776522E-06	9.28367593E-08	6.31818429E-09	8.41121271E-10
21	3.03348271E-06	3.96941158E-08	2.34211283E-09	2.80173310E-10
22	1.61688273E-06	1.69914439E-08	8.69201397E-10	9.34310940E-11
23	8.62721780E-07	7.28094780E-09	3.22913129E-10	3.11894931E-11
24	4.60764715E-07	3.12291546E-09	1.20078423E-10	1.04217049E-11
25	2.46302980E-07	1.34064315E-09	4.46914351E-11	3.48537021E-12

n	$x_n(t) \ t=5$	$x_n(t) \ t=6$	$x_n(t) \ t=7$	$x_n(t) \ t=8$
2	6.37060137E-01	5.96724973E-01	5.63796491E-01	5.36192781E-01
3	1.61291554E-01	1.40725238E-01	1.25060596E-01	1.12695687E-01
4	4.35866856E-02	3.54014422E-02	2.95780192E-02	2.52454608E-02
5	1.21867597E-02	9.21277514E-03	7.23574004E-03	5.84898215E-03
6	3.48035794E-03	2.44867389E-03	1.80777483E-03	1.38390400E-03
7	1.00845016E-03	6.60313982E-04	4.58218409E-04	3.32191565E-04
8	2.95302928E-04	1.79946546E-04	1.17372689E-04	8.05808180E-05
9	8.71702865E-05	4.94331154E-05	3.03066722E-05	1.97037023E-05
10	2.58948143E-05	1.36657041E-05	7.87491621E-06	4.84839385E-06
11	7.73165277E-06	3.79715547E-06	2.05666584E-06	1.19910234E-06
12	2.31823858E-06	1.05951932E-06	5.39391400E-07	2.97807597E-07
13	6.97550235E-07	2.96680737E-07	1.41962299E-07	7.42237705E-08
14	2.10521185E-07	8.33242726E-08	3.74750979E-08	1.85545412E-08
15	6.36998096E-08	2.34625247E-08	9.91819840E-09	4.65025828E-09
16	1.93177616E-08	6.62145605E-09	2.63086000E-09	1.16809554E-09
17	5.86992823E-09	1.87235819E-09	6.99228074E-10	2.93991937E-10
18	1.78676571E-09	5.30374530E-10	1.86164922E-10	7.41224925E-11
19	5.44726899E-10	1.50470836E-10	4.96423629E-11	1.87171589E-11
20	1.66301720E-10	4.27492335E-11	1.32560175E-11	4.73299051E-12
21	5.08348939E-11	1.21604955E-11	3.54421766E-12	1.19833385E-12
22	1.55569070E-11	3.46314031E-12	9.48687893E-13	3.03749453E-13
23	4.76580727E-12	9.87280470E-13	2.54201312E-13	7.70733947E-14
24	1.46137941E-12	2.81724154E-13	6.81781376E-14	1.95752022E-14
25	4.48506693E-13	8.04612686E-14	1.83017066E-14	4.97607616E-15